APPLICATION OF THE METHOD OF ASYMPTOTIC INTEGRATION TO THE CONSTRUCTION OF AN APPROXIMATE THEORY OF ANISOTROPIC SHELLS

(PRIMENENIE METODA ASIMPTOTICHESKOGO INTEGRIROVANIIA K POSTROENIIU PRIBLIZHENNOI TEORII ANIZOTROPNYKH OBOLOCHEK)

PMM Vol.30, № 2, 1966, pp.388-398

L.A.AGALOVIAN

(Yerevan)

(Received March 30, 1965)

The investigation deals with shells with general anisotropy without the assumptions which are generally made in deriving the basic equations of classical shell theory. In the case of general anisotropy, it is assumed that there is only one plane of elastic symmetry at each point, namely the plane which is parallel to the middle surface of the shell.

Using the method of asymptotic integration proposed by Gol'denveizer [1 and 2], it is shown that the state of stress in an anisotropic shell may be expressed as the sum of two stress states. The first is defined by the equations which are obtained from the fundamental iterative process; the second is derived from an auxiliary iterative process.

1. Fundamental equations of anisotropic elasticity. Some of the results of classical linear elasticity [3], expressed in terms of general curvilinear coordinates ϑ^i, are introduced here for subsequent use.

A point in the shell is represented by the position vector

$$\mathbf{R} = \mathbf{r}(\vartheta^1, \vartheta^2) + \vartheta^3 \mathbf{n}(\vartheta^1, \vartheta^2) \tag{1.1}$$

where the vector $\mathbf{r}(\vartheta^1, \vartheta^2)$ pertains to the position in the middle surface of the shell in terms of the curvilinear coordinates ϑ^α; $\mathbf{n}$ is the unit normal to the middle surface associated with the ϑ^3 direction; $-h \leqslant \vartheta^3 \leqslant +h$, the shell thickness $2h$ is assumed constant. The metric tensors g_{ij} and g^{ij} as well as the unit tensor δ^i_j are expressed in terms of the basic covariant and contravariant vectors $\mathbf{g}_i$ and $\mathbf{g}^i$, respectively, by Formulas

$$g_{ij} = \mathbf{g}_i \mathbf{g}_j, \qquad g^{ij} = \mathbf{g}^i \mathbf{g}^j, \qquad \delta_j^{\ i} = g^{ir} g_{rj}$$

Let $a_{\alpha\beta}$ and $b_{\alpha\beta}$ be, respectively, the tensors of the first and second quadratic forms of the middle surface. Then the following relations hold among the various geometric quantities which characterize the shell and its middle surface [3]:

$$\mathbf{g}_\alpha = \partial \mathbf{R} / \partial \vartheta^\alpha = (\delta_\alpha^{\ \lambda} - \vartheta^3 b_\alpha^{\ \lambda})\, \mathbf{a}_\lambda, \qquad \mathbf{g}_3 = \partial \mathbf{R} / \partial \vartheta^3 = \mathbf{n} \tag{1.2}$$

$$g_{\alpha\beta} = a_{\alpha\beta} - 2\vartheta^3 b_{\alpha\beta} + (\vartheta^3)^2 b_\alpha^{\ \lambda} b_{\beta\lambda}, \qquad g_{\alpha 3} = 0, \; g_{33} = 1 \tag{1.3}$$

$$a_{\alpha\beta} = \mathbf{a}_\alpha \mathbf{a}_\beta, \qquad g = |g_{ij}|, \qquad a = |a_{\alpha\beta}| \tag{1.4}$$

$$\vartheta = 1 - \vartheta^3 b_\lambda^{\ \lambda} + (\vartheta^3)^2 K, \qquad K = b_1^{\ 1} b_2^{\ 2} - b_2^{\ 1} b_1^{\ 2} \qquad (\vartheta = \sqrt{g / a})$$

Here K is the Gaussian curvature of the middle surface; b_α^λ is the mixed form of the second fundamental tensor $b_{\alpha\beta}$. Here and hereinafter Greek indices range over the values 1 and 2 whereas Latin indices range over the values 1, 2 and 3.

Let us introduce the covariant strain tensor γ_{ij} and the contravariant stress tensor σ^{ij}. The first is expressible in terms of the displacement vector $\mathbf{U}$ as follows:

$$\gamma_{ij} = {}^1\!/_2 (\mathbf{g}_i \, \partial \mathbf{U} / \partial \vartheta^j + \mathbf{g}_j \, \partial \mathbf{U} / \partial \vartheta^i) \tag{1.5}$$

In absence of body forces, the equilibrium equations take the form

$$\partial \mathbf{T}^i / \partial \vartheta^i = 0 \qquad (\mathbf{T}^i = \sqrt{g}\, \sigma^{ij} \mathbf{g}_j) \tag{1.6}$$

It is convenient to introduce here the asymmetric stress tensor τ^{ij}

$$\mathbf{T}_i = (\tau^{i\lambda} \mathbf{a}_\lambda + \tau^{i3} \mathbf{n}) \sqrt{a}, \qquad \tau^{i\lambda} = (\delta_\mu^{\ \lambda} - \vartheta^3 b_\mu^{\ \lambda}) \sigma^{i\mu} \vartheta, \qquad \tau^{i3} = \vartheta \sigma^{i3} \tag{1.7}$$

The combination of (1.6) and (1.7) yields

$$\vartheta \sigma_\beta^{\ i} = \tau^{i\lambda} (a_{\lambda\beta} - \vartheta^3 b_{\lambda\beta}), \qquad \sigma_3^{\ i} \vartheta = \tau^{i3} = \vartheta \sigma^{i3} \tag{1.8}$$

In view of the symmetry of σ^{ij}, we have

$$c_{\lambda\beta} (\tau^{\lambda\beta} - \vartheta^3 b_\alpha^\lambda \tau^{\alpha\beta}) = 0, \qquad \tau^{3\lambda} = \tau^{\lambda 3} - \vartheta^3 b_\mu^\lambda \tau^{\mu 3} \tag{1.9}$$

Here $c_{\lambda\beta}$ is an antisymmetric tensor with components

$$c_{11} = c_{22} = 0, \qquad c_{12} = -c_{21} = \sqrt{a} \tag{1.10}$$

Taking into account (1.7), the equilibrium equations (1.6) may be transformed into

$$\nabla_\alpha \tau^{\alpha\beta} - b_\alpha^{\ \beta} \tau^{\alpha 3} + \partial \tau^{3\beta} / \partial \vartheta^3 = 0, \qquad \nabla_\alpha \tau^{\alpha 3} + b_{\alpha\beta} \tau^{\alpha\beta} + \partial \tau^{33} / \partial \vartheta^3 = 0 \tag{1.11}$$

where ∇_α represents covariant differentiation in the metric of the middle surface [4]

$$\nabla_\lambda A^{\alpha\beta} = \partial A^{\alpha\beta} / \partial \vartheta^\lambda + \Gamma_{\mu\lambda}^{\alpha} A^{\mu\beta} + \Gamma_{\mu\lambda}^{\ \beta} A^{\alpha\mu}$$

$$\Delta_\lambda A_\alpha = \partial A_\alpha / \partial \vartheta^\lambda - \Gamma_{\alpha\lambda}^{\ \mu} A_\mu, \qquad \nabla_\lambda A^\alpha = \partial A^\alpha / \partial \vartheta^\lambda + \Gamma_{\mu\lambda}^{\alpha} A^\mu, \qquad \nabla_\lambda A = \partial A / \partial \vartheta^\lambda \tag{1.12}$$

$$\Gamma_{\beta\gamma}^{\ \alpha} = a_\alpha \partial a_\beta / \partial \vartheta^\gamma = {}^1\!/_2 a^{\alpha\lambda} (\partial a_{\beta\lambda} / \partial \vartheta^\gamma + \partial a_{\gamma\lambda} / \partial \vartheta^\beta - \partial a_{\beta\gamma} / \partial \vartheta^\lambda) \tag{1.13}$$

We now write the displacement vector $\mathbf{U}$ in the form

$$\mathbf{U} = u^\lambda \mathbf{a}_\lambda - W \mathbf{n} \tag{1.14}$$

Taking into account (1.5), we obtain

$$\begin{gathered} 2\gamma_{\alpha\beta} = \nabla_\alpha u_\beta + \nabla_\beta u_\alpha + 2 b_{\alpha\beta} W - \vartheta^3 [b_\beta^\lambda (\nabla_\alpha u_\lambda + b_{\lambda\alpha} W) + b_\alpha^\lambda (\nabla_\beta u_\lambda + b_{\lambda\beta} W)] \\ 2\gamma_{\alpha 3} = -\nabla_\alpha W + b_\alpha^{\ \lambda} u_\lambda + \partial u_\alpha / \partial \vartheta^3 - \vartheta^3 b_\alpha^{\ \lambda} \partial u_\lambda / \partial \vartheta^3, \quad \gamma_{33} = -\partial W / \partial \vartheta^3 \end{gathered} \tag{1.15}$$

In general curvilinear coordinates, the stress-strain relations [3] are given by

$$\gamma_{ij} = F_{ijrs} \sigma^{rs} \quad \text{for} \quad \gamma_{ij} = F_{ij}^{\ rs} \sigma_r^k g_{ks} \tag{1.16}$$

Here, the elastic constants F_{ij}^{rs} in an arbitrary curvilinear coordinate system are related to the constants S_{ij}^{rs} of an orthogonal coordinate system by the relations

$$F_{ij}^{\ rs} = \frac{\partial x^p}{\partial \vartheta^i} \frac{\partial x^q}{\partial \vartheta^j} \frac{\partial \vartheta^r}{\partial x^m} \frac{\partial \vartheta^s}{\partial x^n} S_{pq}^{\ mn} \tag{1.17}$$

Let us consider an anisotropic elastic body having one plane of symmetry at each point (13 independent constants). Assume that the plane of elastic symmetry is everywhere parallel to the plane $\vartheta^3 = x^3 = \text{const}$.

Then

$$F_{ij}^{\ rs} = F_{ij}^{\ sr} = F_{ji}^{\ rs} = F_{ji}^{\ sr}, \qquad F_{\lambda\mu}^{\ 3\beta} = F_{3\beta}^{\ \lambda\mu} = F_{3\mu}^{\ 33} = F_{33}^{\ 3\mu} = 0 \tag{1.18}$$

The relations (1.16) may be transformed with the aid of (1.18) and (1.3) into

$$\gamma_{\alpha\beta} = F_{\alpha\beta}^{\ \lambda\mu} \sigma_\lambda^\xi g_{\xi\mu} + F_{\alpha\beta}^{\ 33} \sigma_3^3, \quad \gamma_{\alpha 3} = F_{\alpha 3}^{\ 3\lambda} \sigma_3^\xi g_{\xi\mu} + F_{\alpha 3}^{\ \lambda 3} \sigma_\lambda^3, \quad \gamma_{33} = F_{33}^{\ \lambda\mu} \sigma_\lambda^\xi g_{\xi\mu} + F_{33}^{\ 3} \sigma_3^3 \tag{1.19}$$

Substituting (1.8) into (1.19), we obtain

$$\vartheta\gamma_{\alpha\beta} = F_{\alpha\beta}^{\ \lambda\mu} g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda} - \vartheta^3 b_{\nu\lambda}) + F_{\alpha\beta}^{\ 33}\tau^{33}, \quad \vartheta\gamma_{\alpha3} = F_{\alpha3}^{\ \lambda3}\tau^{\xi3} g_{\xi\lambda} + F_{\alpha3}^{\ 3\lambda}\tau^{3\xi}(a_{\xi\lambda} - \vartheta^3 b_{\xi\lambda}) \tag{1.20}$$

$$\vartheta\gamma_{33} = F_{33}^{\ \lambda\mu} g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda} - \vartheta^3 b_{\nu\lambda}) + F_{33}^{\ 33}\tau^{33}$$

In particular, for an isotropic body

$$F_{ij}^{\ rs} = [(1+\sigma)/E]\,\delta_i^r\delta_j^s - \sigma/E g^{rs} g_{ij} \tag{1.21}$$

Here E is Young's modulus, and σ is Poisson's ratio. Thus, (1.20) is reduced to the well known relations for an isotropic body.

With the aid of (1.15), (1.20) may be rewritten in the following form:

$$-\vartheta\partial W/\partial\vartheta^3 = F_{33}^{\ \lambda\mu} g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda} - \vartheta^3 b_{\nu\lambda}) + F_{33}^{\ 33}\tau^{33}$$

$$^1/_2\vartheta(-\nabla_\alpha W + b_\alpha^\lambda u_\lambda + \partial u_\alpha/\partial\vartheta^3 - \vartheta^3 b_\alpha^\lambda \partial u_\lambda/\partial\vartheta^3) = \tag{1.22}$$

$$= F_{\alpha3}^{\ \lambda3}\tau^{\xi3} g_{\xi\lambda} + F_{\alpha3}^{\ 3\lambda}\tau^{3\xi}(a_{\xi\lambda} - \vartheta^3 b_{\xi\lambda})$$

$$^1/_2\vartheta\{\nabla_\alpha u_\beta + \nabla_\beta u_\alpha + 2b_{\alpha\beta}W - \vartheta^3[b_\beta^\lambda(\nabla_\alpha u_\lambda + b_{\lambda\alpha}W) +$$

$$+ b_\alpha^\lambda(\nabla_\beta u_\lambda + b_{\lambda\beta}W)]\} = F_{\alpha\beta}^{\ \lambda\mu} g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda} - \vartheta^3 b_{\nu\lambda}) + F_{\alpha\beta}^{\ 33}\tau^{33}$$

2. Transformation of the fundamental relations. The equilibrium equations (1.11), together with symmetry relations (1.9) and stress-strain relations (1.22) constitute a complete system of differential equations, defining displacements and stresses.

To integrate this system, let us introduce a new system of independent variables defined by

$$\vartheta^\alpha = R\xi^\alpha, \qquad \vartheta^3 = h\zeta \tag{2.1}$$

Here R is a characteristic radius of curvature of the middle surface, and $2h$ is the shell thickness. We will assume that the state of stress varies rapidly as a function of ϑ^3, only, whereas the variation of stresses and displacements as functions of ξ^1, ξ^2 and ζ is not too rapid. Then (1.9), (1.11) and (1.22) take the form

$$h^*\nabla_\alpha'\tau^{\alpha\beta} - h^*Rb_\alpha^{\ \beta}\tau^{\alpha3} + \partial\tau^{3\beta}/\partial\zeta = 0, \quad h^*\nabla_\alpha'\tau^{\alpha3} + h^*Rb_{\alpha\beta}\tau^{\alpha\beta} + \partial\tau^{33}/\partial\zeta = 0 \tag{2.2}$$

$$c_{\lambda\beta}(\tau^{\lambda\beta} - h^*R\zeta b_\alpha^\lambda\tau^{\alpha\beta}) = 0, \qquad \tau^{3\lambda} = \tau^{\lambda3} - h^*R\zeta b_\mu^\lambda\tau^{\mu3} \qquad (h^* = h/R)$$

Here $\nabla_\alpha' = R\nabla_\alpha$ represents covariant differentiation with respect to ξ^α.

Stress-strain relations. It is clear from (1.17) that the F_{ij}^{rs} depend not only on the physical constants characterizing the mechanical properties of the material, but on the coordinate system as well. For an isotropic material, the physical properties of the material are independent of direction. Therefore, as (1.21) shows, the physical constants in this case appear simply as scalar multipliers, not involving the metric tensor. This fact simplifies calculations considerably, since, upon transformation into a new coordinate system by means of (2.1), the dependence of various quantities on the small parameter h^* is immediately clear, provided that the metric tensor is known. In the anisotropic case, on the other hand, the situation is different. Here, the physical and geometric directions are not aligned with each other, since the physical properties vary with direction, so that (1.17) can no longer be written in the form of (1.21) or some similar form in which the physical and geometrical properties would be separated. While this makes computations considerably more difficult, the difficulty is not insurmountable. As (1.17) shows, the quantities F_{ij}^{rs}, obtained in transforming into a new coordinate system will contain the small parameter h^*. Let us expand these quantities in power series of h^* (in many cases these series may have a finite number of terms), i.e. let

$$F_{\alpha\beta}^{\ \lambda\mu} = \sum_{s=0}^{s=S} h^{*s} F_{\alpha\beta}^{\ \lambda\mu(s)}, \quad F_{\alpha\beta}^{\ 33} = \sum_{s=0}^{s=S} h^{*s} F_{\alpha\beta}^{\ 33(s)}$$

$$F_{33}^{\ \alpha\beta} = \sum_{s=0}^{s=S} h^{*s} F_{33}^{\ \alpha\beta(s)}, \quad F_{\alpha3}^{\ 3\beta} = \sum_{s=0}^{s=S} h^{*s} F_{\alpha3}^{\ 3\beta(s)} \tag{2.3}$$

Utilizing (2.1) and (2.3), (1.22) may be written as

$$-\frac{\vartheta}{R}\frac{\partial W}{\partial\zeta}=h^*\Big[\sum_{s=0}^{s=S}h^{*s}F_{33}^{33(s)}\tau^{33}+\sum_{s=0}^{s=S}h^{*s}F_{33}^{\lambda\mu(s)}g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda}-h^*\zeta Rb_{\nu\lambda})\Big]$$

$$\tfrac{1}{2}R^{-1}\vartheta\,(-h^*\nabla_\alpha{}'W+h^*Rb_\alpha{}^\lambda u_\lambda+\partial u_\alpha/\partial\zeta-h^*\zeta Rb_\alpha^\lambda\partial u_\lambda/\partial\zeta)=$$

$$=h^*\Big[\sum_{s=0}^{s=S}h^{*s}F_{\alpha3}^{\lambda3(s)}\tau^{\xi3}g_{\xi\lambda}+\sum_{s=0}^{s=S}h^{*s}F_{\alpha3}^{3\lambda(s)}\tau^{3\xi}(a_{\xi\lambda}-h^*R\zeta b_{\xi\lambda})\Big]\qquad(2.4)$$

$$\tfrac{1}{2}R^{-1}\vartheta\,\{\nabla_\beta{}'u_\alpha+\nabla_\alpha{}'u_\beta+2Rb_{\alpha\beta}W-\zeta Rh^*[b_\beta^\lambda(\nabla_\alpha{}'u_\lambda+Rb_{\lambda\alpha}W)+$$

$$+b_\alpha^\lambda(\nabla_\beta{}'u_\lambda+Rb_{\lambda\beta}W)]\}=\sum_{s=0}^{s=S}h^{*s}F_{\alpha\beta}^{\lambda\mu(s)}g_{\xi\mu}\tau^{\xi\nu}(a_{\nu\lambda}-h^*R\zeta b_{\nu\lambda})+\sum_{s=0}^{s=S}h^{*s}F_{\alpha\beta}^{33(s)}\tau^{33}$$

Here, in accordance with (1.3), (1.4) and (2.1), $g_{\alpha\beta}$ and ϑ are given as

$$g_{\alpha\beta}=a_{\alpha\beta}-2h^*R\zeta b_{\alpha\beta}+h^{*2}\zeta^2R^2b_\alpha^\lambda b_{\beta\lambda},\quad \vartheta=1-h^*\xi Rb_\lambda^\lambda+h^{*2}\zeta^2R^2K\qquad(2.5)$$

B o u n d a r y c o n d i t i o n s . Assume that there are applied stresses $\tau^{3\alpha}$ and τ^{33}, acting on the external and internal shell surfaces $\vartheta^3=\pm h$ and that, per unit area of the shell's middle surface, these stresses are

$$\tau^{33}=\pm\tfrac{1}{2}p,\qquad \tau^{3\alpha}=\pm\tfrac{1}{2}P^\alpha\qquad(2.6)$$

3. Fundamental iterative process. The iterative process used was developed by Gol'denveizer in [1 and 2].

The procedure which permits the determination of the fundamental stresses, i.e. those stresses which do not generally attenuate rapidly with distance from the shell boundaries, will be termed the fundamental iterative process. Let Q be some stress component, and V some displacement component. We will seek solutions to (2.2) and (2.4) in the form

$$Q=\frac{1}{h^{*r}}\sum_{s=0}^{s=S}h^{*s}Q_{(s)},\qquad V=\frac{1}{h^{*r}}\sum_{s=0}^{s=S}h^{*s}V^{(s)}\qquad(3.1)$$

Here it is assumed that $Q_{(s)}\equiv0, V^{(s)}\equiv0$ for $s<0$; ; the integers r are different for different stress and displacement components. The various r must be chosen after substituting (3.1) into (2.2) and (2.4). The choice must be such that equating to zero the coefficients of like powers of h^* yields a consistent sequence of systems of equations for the coefficients in the power series (3.1). Such a set of r is referred to as a consistent set.

We choose the r as follows (the integer $\varkappa$ is as yet undetermined):

$$\tau^{\alpha\beta}\to r=\varkappa+1,\qquad (\tau^{\alpha3},\tau^{33})\to r=\varkappa,\qquad (u_\alpha, W)\to r=\varkappa+1\qquad(3.2)$$

Substitution of (3.1) into (2.2) and (2.4) and taking into account (3.2), yields the following system of equations for the determination of the power series coefficients in (3.1):

$$\nabla_\alpha{}'\tau_{(s)}^{\alpha\beta}-b_\alpha^\beta R\tau_{(s-1)}^{\alpha3}+\partial\tau_{(s)}^{3\beta}/\partial\zeta=0,\ \ \nabla_\alpha{}'\tau_{(s-1)}^{\alpha3}+Rb_{\alpha\beta}\tau_{(s)}^{\alpha\beta}+\partial\tau_{(s)}^{33}/\partial\zeta=0$$

$$c_{\lambda\beta}[\tau_{(s)}{}^{\lambda\beta}-R\zeta b_\mu^\lambda\tau_{(s-1)}^{\mu3}]=0,\qquad \tau_{(s)}{}^{3\lambda}=\tau_{(s)}{}^{\lambda3}-R\zeta b_\mu^\lambda\tau_{(s-1)}^{\mu3}\qquad(3.3)$$

$$-R^{-1}\partial W^{(s)}/\partial\zeta+\zeta b_\lambda^\lambda\partial W^{(s-1)}/\partial\zeta-\zeta^2RK\partial W^{(s-2)}/\partial\zeta=F_{33}^{33(0)}\tau_{(s-2)}^{33}+$$

$$+F_{33}^{33(1)}\tau_{(s-3)}^{33}+\ldots+F_{33}^{33(k)}\tau_{(s-k-2)}^{33}+\ldots+F_{33}^{\lambda\mu(0)}[a_{\xi\mu}a_{\nu\lambda}\tau_{(s-1)}^{\xi\nu}-$$

$$-\zeta R(a_{\xi\mu}b_{\nu\lambda}+2a_{\nu\lambda}b_{\xi\mu})\tau_{(s-2)}^{\xi\nu}+\zeta^2R^2(2b_{\xi\mu}b_{\nu\lambda}+a_{\nu\lambda}b_\xi^\alpha b_{\mu\alpha})\tau_{(s-3)}^{\xi\nu}-$$

$$-\zeta^3R^3b_\xi^\alpha b_{\mu\alpha}b_{\nu\lambda}\tau_{(s-4)}^{\xi\nu}]+\ldots+F_{33}^{\lambda\mu(k)}[\ldots]_{(s-k)}+\ldots$$

$$\tfrac{1}{2}R^{-1}[-\nabla_\alpha' W^{(s-1)} + Rb_\alpha^\lambda u_\lambda^{(s-1)} + \partial u_\alpha^{(s)}/\partial\zeta - \zeta R b_\alpha^\lambda \partial u_\lambda^{(s-1)}/\partial\zeta] - \tag{3.4}$$
$$-\tfrac{1}{2}\zeta b_\lambda^\lambda[\ldots]_{(s-1)} + \tfrac{1}{2}\zeta^2 RK[\ldots]_{(s-2)} = F_{\alpha 3}^{\lambda 3(0)}[a_{\xi\lambda}\tau_{(s-2)}^{\xi 3} - 2\zeta R b_{\xi\lambda}\tau_{(s-3)}^{\xi 3} +$$
$$+\zeta^2 R^2 b_\xi^\nu b_{\lambda\nu}\tau_{(s-4)}^{\xi 3}] + F_{\alpha 3}^{3\lambda(0)}[a_{\xi\lambda}\tau_{(s-2)}^{3\xi} - R\zeta b_{\xi\lambda}\tau_{(s-3)}^{3\xi}] + \ldots + F_{\alpha 3}^{\lambda 3(k)}[\ldots]_{(s-k)} + \ldots$$

$$\tfrac{1}{2}R^{-1}\{\nabla_\alpha' u_\beta^{(s)} + \nabla_\beta' u_\alpha^{(s)} + 2Rb_{\alpha\beta}W^{(s)} - \zeta R[b_\beta^\lambda(\nabla_\alpha' u_\lambda^{(s-1)} + Rb_{\lambda\alpha}W^{(s-1)}) +$$
$$+ b_\alpha^\lambda(\nabla_\beta' u_\lambda^{(s-1)} + Rb_{\lambda\beta}W^{(s-1)})]\} - \tfrac{1}{2}b_\lambda^\lambda\{\ldots\}_{(s-1)} + \tfrac{1}{2}\zeta^2 RK\{\ldots\}_{(s-2)} =$$
$$= F_{\alpha\beta}^{\lambda\mu(0)}[a_{\xi\mu}a_{\nu\lambda}\tau_{(s)}^{\xi\nu} - \zeta R(a_{\xi\mu}b_{\nu\lambda} + 2a_{\nu\lambda}b_{\xi\mu})\tau_{(s-1)}^{\xi\nu} +$$
$$+\zeta^2 R^2(2b_{\xi\mu}b_{\nu\lambda} + a_{\nu\lambda}b_\xi^\alpha b_{\mu\alpha})\tau_{(s-2)}^{\xi\nu} - \zeta^3 R^3 b_\xi^{\ \eta} b_{\mu\eta} b_{\nu\lambda}\tau_{(s-3)}^{\xi\nu} + \ldots] +$$
$$+ F_{\alpha\beta}^{\lambda\mu(k)}[\ldots]_{(s-k)} + \ldots + F_{\alpha\beta}^{33(0)}\tau_{(s-1)}^{33} + \ldots + F_{\alpha\beta}^{33(k)}\tau_{(s-k-1)}^{33} + \ldots$$

Here and hereinafter the symbol $[\ldots]_{(s-k)}$, with $k = 1, 2, \ldots$, represents the expression contained in the immediately preceding brackets with s replaced by $(s - k)$.

As one might expect, Equations (3.3) coincide with the corresponding equations for isotropic shells, whereas Equations (3.4) are essentially different from the corresponding isotropic ones. The orthotropic and isotropic shell equations may be obtained as special cases of (3.4).

Setting $F_{12}^{ii} = F_{ii}^{12} = 0$ and $F_{13}^{23} = 0$, in (3.4), leads to the corresponding orthotropic shell equations. If we set

$$F_{\alpha\beta}^{\lambda\mu(0)} = [(1+\sigma)/E]\,\delta_\alpha^\lambda\delta_\beta^\mu - (\sigma/E)\,a^{\lambda\mu}a_{\alpha\beta}$$

in (3.5), a result which may be obtained by neglecting terms containing h^* in (1.21), we obtain an equation which coincides with the corresponding isotropic shell equation [1].

The leading system of equations in (3.3) and (3.4) is given by

$$\nabla_\alpha'\tau_{(0)}^{\alpha\beta} + \partial\tau_{(0)}^{3\beta}/\partial\zeta = 0, \quad Rb_{\alpha\beta}\tau_{(0)}^{\alpha\beta} + \partial\tau_{(0)}^{33}/\partial\zeta = 0, \quad c_{\lambda\beta}\tau_{(0)}^{\lambda\beta} = 0, \quad \tau_{(0)}^{3\lambda} = \tau_{(0)}^{\lambda 3}$$
$$\partial W^{(0)}/\partial\zeta = 0, \quad \partial u_\alpha^{(0)}/\partial\zeta = 0 \tag{3.5}$$
$$\tfrac{1}{2}R^{-1}[\nabla_\alpha' u_\beta^{(0)} + \nabla_\beta' u_\alpha^{(0)} + 2Rb_{\alpha\beta}W^{(0)}] = F_{\alpha\beta}^{\lambda\mu(0)}a_{\xi\mu}a_{\nu\lambda}\tau_{(0)}^{\xi\nu}$$

By making use of boundary conditions (2.6), Equations (3.5) are readily integrable with respect to ζ. For this purpose, (2.6) will be written in the form

$$P^\alpha = \sum_{s=0}^{s=S} h^{*s}P_{(s)}^{\alpha}, \qquad p = \sum_{s=0}^{s=S} h^{*s}p_{(s)} \tag{3.6}$$

Integration of (3.5) with respect to ζ then yields

$$W^{(0)} = w^{(0)}(\xi^1, \xi^2), \qquad u_\alpha^{(0)} = v_\alpha^{(0)}(\xi^1, \xi^2), \qquad c_{\lambda\beta}\tau_{(0)}^{\lambda\beta} = 0, \qquad \tau_{(0)}^{3\lambda} = \tau_{(0)}^{\lambda 3}$$
$$\nabla_\alpha'\tau_{(0)}^{\alpha\beta} = -\tfrac{1}{2}P_{(0)}^{\beta}, \qquad Rb_{\alpha\beta}\tau_{(0)}^{\alpha\beta} = -\tfrac{1}{2}p_{(0)}, \quad \tau^{3\beta}{}_{(0)} = \tfrac{1}{2}\zeta P_{(0)}^\beta, \qquad \tau_{(0)}^{33} = \tfrac{1}{2}\zeta p_{(0)}$$
$$\tfrac{1}{2}R^{-1}[\nabla_\alpha' v_\beta^{(0)} + \nabla_\beta' v_\alpha^{(0)} + 2Rb_{\alpha\beta}W^{(0)}] = F_{\alpha\beta}^{\lambda\mu(0)}a_{\xi\mu}a_{\nu\lambda}\tau_{(0)}^{\xi\nu} \tag{3.7}$$

Equations (3.7) comprise a complete system of differential equations independent of ζ and containing ξ^1 and ξ^2 as the independent variables, with $\tau_{(0)}^{\lambda\beta}$, $\tau_{(0)}^{3\lambda}$, $w^{(0)}$ and $v_\alpha^{(0)}$, as the unknown functions. It is apparent from (3.7) that the stresses $\tau_{(0)}^{\alpha\beta}$ do not vary over the shell thickness. Such a state of stress is closely related to the membrane state of stress of classical anisotropic shell theory [5].

Consider Equations (2.2) and (2.4) with the homogeneous boundary conditions

$$P_{(s)}^{\alpha} = p_{(s)} = 0 \tag{3.8}$$

It may be directly verified that in this case there exists another form of the series of expansion (3.1) with the following consistent set of values of r :

$$\tau^{\alpha\beta} \to r = \varkappa + 1, \qquad (\tau^{\alpha 3}, \tau^{33}) \to r = \varkappa, \qquad (u_\alpha, W) \to r = \varkappa + 2 \tag{3.9}$$

Substituting (3.1) and (3.9) into (2.2) and (2.4) and requiring that the sums of coefficients of like powers of h^* vanish, we obtain again Equations (3.3), but (3.4) is replaced by the following equations:

$$-R^{-1}\partial W^{(s)}/\partial\zeta+\zeta b_\lambda^\lambda\partial W^{(s-1)}/\partial\zeta-\zeta^2RK\partial W^{(s-2)}/\partial\zeta=F_{33}^{33(0)}\tau_{(s-3)}^{33}+\ldots$$
$$\ldots+F_{33}^{33(k)}\tau_{(s-3-k)}^{33}+\ldots+F_{33}^{\lambda\mu(0)}[a_{\xi\mu}a_{\nu\lambda}\tau_{(s-2)}^{\xi\nu}-\zeta R(a_{\xi\mu}b_{\nu\lambda}+2a_{\nu\lambda}b_{\xi\mu})\tau_{(s-3)}^{\xi\nu}+$$
$$+\zeta^2R^2(2b_{\xi\mu}b_{\nu\lambda}+a_{\nu\lambda}b_\xi^\alpha b_{\mu\alpha})\tau_{(s-4)}^{\xi\nu}-\zeta^3R^3b_\zeta^\alpha b_{\mu\alpha}b_{\nu\lambda}\tau_{(s-5)}^{\xi\nu}]+\ldots+F_{33}^{\lambda\mu(k)}[\ldots]_{(s-k)}+\ldots$$

$$\tfrac{1}{2}R^{-1}[-\nabla_\alpha' W^{(s-1)}+Rb_\alpha^\lambda u_\lambda^{(s-1)}+\partial u_\alpha^{(s)}/\partial\zeta-\zeta Rb_\alpha^\lambda\partial u_\lambda^{(s-1)}/\partial\zeta]-$$
$$-\tfrac{1}{2}\zeta b_\lambda^\lambda[\ldots]_{(s-1)}+\tfrac{1}{2}\zeta^2RK[\ldots]_{(s-2)}=F_{\alpha 3}^{\lambda 3(0)}[a_{\xi\lambda}(\tau_{(s-3)}^{\xi 3}+\tau_{(s-3)}^{3\xi})-$$
$$-\zeta Rb_{\xi\lambda}(2\tau_{(s-4)}^{\xi 3}+\tau_{(s-4)}^{3\xi})+\zeta^2R^2b_\xi^\nu b_{\lambda\nu}\tau_{(s-5)}^{\xi 3}]+\ldots+F_{\alpha 3}^{\lambda 3(k)}[\ldots]_{(s-k)}+\ldots \qquad (3.10)$$
$$\tfrac{1}{2}R^{-1}\{\nabla_\alpha' u_\beta^{(s)}+\nabla_\beta' u_\alpha^{(s)}+2Rb_{\alpha\beta}W^{(s)}-\zeta R[b_\beta^\lambda(\nabla_\alpha' u_\lambda^{(s-1)}+Rb_{\lambda\alpha}W^{(s-1)})+$$
$$+b_\alpha^\lambda(\nabla_\beta' u_\lambda^{(s-1)}+Rb_{\lambda\beta}W^{(s-1)})]\}-\tfrac{1}{2}\zeta b_\lambda^\lambda\{\ldots\}_{(s-1)}+\tfrac{1}{2}\zeta^2RK\{\ldots\}_{(s-2)}=$$
$$=F_{\alpha\beta}^{\lambda\mu(0)}[a_{\xi\mu}a_{\nu\lambda}\tau_{(s-1)}^{\xi\nu}-\zeta R(a_{\xi\mu}b_{\nu\lambda}+2a_{\nu\lambda}b_{\xi\mu})\tau_{(s-2)}^{\xi\nu}+$$
$$+\zeta^2R^2(2b_{\xi\mu}b_{\nu\lambda}+a_{\nu\lambda}b_\xi^\alpha b_{\mu\alpha})\tau_{(s-3)}^{\xi\nu}-\zeta^3R^3b_\xi^\eta b_{\mu\eta}b_{\nu\lambda}\tau_{(s-4)}^{\xi\nu}]+\ldots$$
$$\ldots+F_{\alpha\beta}^{\lambda\mu(k)}[\ldots]_{(s-k)}+\ldots+F_{\alpha\beta}^{33(0)}\tau_{(s-2)}^{33}+\ldots+F_{\alpha\beta}^{33(k)}\tau_{(s-2-k)}^{33}+\ldots$$

Consider the zeroth approximation in (3.3) and the zeroth and first approximations in (3.10)

$$\nabla_\alpha'\tau_{(0)}^{\alpha\beta}+\partial\tau_{(0)}^{3\beta}/\partial\zeta=0,\qquad Rb_{\alpha\beta}\tau_{(0)}^{\alpha\beta}+\partial\tau_{(0)}^{33}/\partial\zeta=0,\qquad c_{\lambda\beta}\tau_{(0)}^{\lambda\beta}=0$$
$$\tau_{(0)}^{3\lambda}=\tau_{(0)}^{\lambda 3},\quad \partial W^{(0)}/\partial\zeta=0,\quad \partial u_\alpha^{(0)}/\partial\zeta=0,\qquad \nabla_\beta' u_\alpha^{(0)}+\nabla_\alpha' u_\beta^{(0)}+2Rb_{\alpha\beta}W^{(0)}=0$$
$$\partial W^{(1)}/\partial\zeta=0,\qquad \partial u_\alpha^{(1)}/\partial\zeta-\nabla_\alpha' W^{(0)}+Rb_\alpha^{\ \lambda}u_\lambda^{(0)}=0 \qquad (3.11)$$
$$\tfrac{1}{2}R^{-1}\{\nabla_\alpha' u_\beta^{(1)}+\nabla_\beta' u_\alpha^{(1)}+2Rb_{\alpha\beta}W^{(1)}-\zeta R[b_\beta^\lambda(\nabla_\alpha' u_\lambda^{(0)}+Rb_{\lambda\alpha}W^{(0)})+$$
$$+b_\alpha^\lambda(\nabla_\beta' u_\lambda^{(0)}+Rb_{\lambda\beta}W^{(0)})]\}=F_{\alpha\beta}^{\lambda\mu(0)}a_{\xi\mu}a_{\nu\lambda}\tau_{(0)}^{\xi\nu}$$

It is clear from (3.11) that $\tau_{(0)}^{\alpha\beta}$ is a linear function of ζ, so that $\tau_{(0)}^{\alpha\beta}=\zeta a(\xi^1,\xi^2)+b(\xi^1,\xi^2)$. Integrating (3.11) and taking into account (3.8), we obtain $b=0$, i.e. $\tau_{(0)}^{\alpha\beta}$ is a homogeneous function of ζ. Note that further integration of (3.11) and application of boundary conditions (3.8) yields

$$W^{(0)}=w^{(0)}(\xi^1,\xi^2),\qquad u_\alpha^{(0)}=v_\alpha^{(0)}(\xi^1,\xi^2),\qquad \nabla_\beta' v_\alpha^{(0)}+\nabla_\alpha' v_\beta^{(0)}+2Rb_{\alpha\beta}w^{(0)}=0$$
$$W^{(1)}=w^{(1)}(\xi^1,\xi^2),\qquad u_\alpha^{(1)}=\zeta(\nabla_\alpha W^{(0)}-Rb_\alpha^{\ \lambda}v_\lambda^{(0)}) \qquad (3.12)$$
$$\tfrac{1}{2}\zeta R^{-1}[\nabla_\alpha'(\nabla_\beta' w^{(0)}-Rb_\beta^\lambda v_\lambda^{(0)})+\nabla_\beta(\nabla_\alpha' w^{(0)}-Rb_\alpha^\lambda v_\lambda^{(0)})-$$
$$-Rb_\beta^\lambda(\nabla_\alpha' u_\lambda^{(0)}+Rb_{\lambda\alpha}w^{(0)})-Rb_\alpha^{\ \lambda}(\nabla_\beta' u_\alpha^{(0)}+Rb_{\lambda\beta}w^{(0)})]=F_{\alpha\beta}^{\lambda\mu(0)}a_{\xi\mu}a_{\nu\lambda}\tau_{(0)}^{\xi\nu}$$
$$\tau_{(0)}^{3\beta}=\tfrac{1}{2}(1-\zeta^2)\nabla_\alpha'\tau_{(0)}^{\alpha\beta}/\zeta,\qquad \tau_{(0)}^{33}=\tfrac{1}{2}(1-\zeta^2)Rb_{\alpha\beta}\tau_{(0)}^{\alpha\beta}/\zeta \qquad (3.13)$$

By comparison with the analogous isotropic shell cases it is easily seen that Equations (3.12) and (3.13) define a state of stress which is closely related to the pure membrane state of stress of classical anisotropic shell theory.

It is also important to note that in the works of Ambartsumian and others [5], the form of the stresses $\tau^{3\alpha}$, as given in (3.13) or in more general form, is assumed; here, it is obtained from asymptotic integration of the equations of elasticity, thus providing proof of the above mentioned assumptions (within the limits of accuracy of those theories).

4. States of stress with a greater degree of variation. Consider states of stress and strain, which vary rapidly as functions of θ^1 and θ^2 in addition to their rapid variation as functions of θ^3. These states will be represented in the forms

$$\vartheta^{\alpha} = \xi^{\alpha} R / K_{(\alpha)}, \qquad \vartheta^{3} = h\zeta \tag{4.1}$$

and we will assume that the variation of stresses and strains as functions of (ξ^1, ξ^2, ζ) is not great. Here, $K_{(\alpha)}$ is a dimensionless constant which is large compared to unity and which increases in magnitude as the variation in the states of stress and strain increases. Following [6], we let $K_{(\alpha)} = (h^*)^{-t_\alpha}$ with $t_\alpha = p_\alpha / q_\alpha$, where t_α is the exponent of the variation in the ϑ^α directions while p_α, and q_α are positive integers. Introducing transformation (4.1) into (1.12), we obtain

$$\begin{gathered} R\nabla_\lambda A^{\alpha\beta} = K_{(\lambda)} \nabla_\lambda^* A^{\alpha\beta}, \qquad R\nabla_\lambda A_\alpha = K_{(\lambda)} \triangle_\lambda^* A_\alpha, \ldots \\ \nabla_\lambda^* A^{\alpha\beta} = \partial A^{\alpha\beta} / \partial \xi^\lambda + (\Gamma_{\mu\lambda}^{\alpha} A^{\mu\beta} + \Gamma_{\mu\lambda}^{\beta} A^{\alpha\mu}) R / K_{(\lambda)} \\ \nabla_\lambda^* A_\alpha = \partial A_\alpha / \partial \xi^\lambda - \Gamma_{\alpha\lambda}^{\mu} A_\mu R / K_{(\lambda)} \end{gathered} \tag{4.2}$$

With the aid of (4.1) and (4.2), Equations (1.11), (1.9) and (1.22) become

$$h^* K_{(\alpha)} \nabla_\alpha^* \tau^{\alpha\beta} - h^* R b_\alpha^{\ \beta} \tau^{\alpha 3} + \frac{\partial \tau^{3\beta}}{\partial \zeta} = 0, \qquad h^* K_{(\alpha)} \nabla_\alpha^* \tau^{\alpha 3} + h^* R b_{\alpha\beta} \tau^{\alpha\beta} + \frac{\partial \tau^{33}}{\partial \zeta} = 0$$

$$c_{\lambda\beta} (\tau^{\lambda\beta} - h^* \zeta R b_\alpha^{\ \lambda} \tau^{\alpha\beta}) = 0, \qquad \tau^{3\lambda} = \tau^{\lambda 3} - h^* \zeta R b_\mu^\lambda \tau^{\mu 3}$$

$$-\frac{\vartheta}{R} \frac{\partial W}{\partial \zeta} = h^* \Big[\sum_{s=0}^{s=S} h^{*s} F_{33}^{\ 33(s)} \tau^{33} + \sum_{s=0}^{s=S} h^{*(s)} F_{33}^{\ \lambda\mu\,(s)} g_{\xi\mu} \tau^{\xi\nu} (a_{\nu\lambda} - h^* \zeta R b_{\nu\lambda}) \Big]$$

$$\begin{aligned} {}^1/_2 R^{-1} \vartheta (-h^* K_{(\alpha)} \nabla_\alpha^* W + h^* R b_\alpha^\lambda u_\lambda + \partial u_\alpha / \partial \zeta - h^* R \zeta b_\alpha^\lambda \partial u_\lambda / \partial \zeta) = \\ = h^* \Big[\sum_{s=0}^{s=S} h^{*s} F_{\alpha 3}^{\ \lambda 3\,(s)} \tau^{\xi 3} g_{\xi\lambda} + \sum_{s=0}^{s=S} h^{*s} F_{\alpha 3}^{\ 3\lambda\,(s)} \tau^{3\xi} (a_{\xi\lambda} - h^* \zeta R b_{\xi\lambda}) \Big] \end{aligned} \tag{4.3}$$

$$\begin{aligned} {}^1/_2 R^{-1} \vartheta \{ K_{(\beta)} \nabla_\beta^* u_\alpha + K_{(\alpha)} \nabla_\alpha^* u_\beta + 2 b_{\alpha\beta} R W - h^* R \zeta [b_\beta^\lambda (K_\alpha \nabla_\alpha^* u_\lambda + R b_{\lambda\alpha} W) + \\ + b_\alpha^\lambda (K_{(\beta)} \nabla_\beta^* u_\lambda + R b_{\lambda\beta} W)] \} = \sum_{s=0}^{s=S} h^{*s} F_{\alpha\beta}^{\ 33\,(s)} \tau^{33} + \sum_{s=0}^{s=S} h^{*s} F_{\alpha\beta}^{\ \lambda\mu\,(s)} g_{\xi\mu} \tau^{\xi\nu} (a_{\nu\lambda} - h^* R \zeta b_{\nu\lambda}) \end{aligned}$$

Consider a state of stress having the same exponent of variation in both coordinate directions. Assume, moreover, that the exponent of variation is nonzero, i.e.

$$K_{(1)} = K_{(2)} = K = h^{*(-t)}, \quad t_{(1)} = t_{(2)} = t = p / q$$

Let $\eta = (h^*)^{-1/q}$. Whence

$$h^* = \eta^{-q}, \qquad K = \eta^p \tag{4.4}$$

Consider a case with large variation, i.e. when $t > \frac{1}{2} (2p > q)$, and assume that $p < q \, (t < 1)$. We seek a solution of (4.3) in the form

$$Q = \eta^r \sum_{s=0}^{s=S} \eta^{-s} Q_{(s)}, \qquad V = \eta^r \sum_{s=0}^{s=S} \eta^{-s} V^{(s)} \tag{4.5}$$

Here, as in (3.1), Q is a typical stress, while V is a typical strain.

In this case there are two different consistent sets of values of r. The first is given by

$$\begin{gathered} \tau^{\alpha\beta} \to r = \varkappa + q, \quad \tau^{\alpha 3} \to r = \varkappa + p, \quad \tau^{33} \to r = \varkappa + 2p - q, \\ u_\alpha \to r = \varkappa + q - p, \qquad W \to r = \varkappa + 2q - 2p \end{gathered} \tag{4.6}$$

The system of equations corresponding to (4.6) is

$$\partial_\alpha \tau_{(s)}{}^{\alpha\beta} + \partial \tau_{(s)}{}^{3\beta} / \partial\zeta - R b_\alpha^\beta \tau_{(s-q)}{}^{\alpha 3} + R\,[\Gamma_{\mu\alpha}^{\alpha} \tau_{(s-p)}{}^{\mu\beta} + \Gamma_{\mu\alpha}^{\beta} \tau_{(s-p)}{}^{\mu\alpha}] = 0 \tag{4.7}$$

$$\partial_\alpha \tau_{(s)}{}^{\alpha 3} + \partial \tau_{(s)}{}^{33} / \partial\zeta + R \Gamma_{\mu\alpha}^{\alpha} \tau_{(s-p)}{}^{\mu 3} + R b_{\alpha\beta} \tau_{(q-2p+s)}^{\alpha\beta} = 0$$

$$c_{\lambda\beta}\,[\tau_{(s)}{}^{\lambda\beta} - \zeta R b_\alpha^\lambda \tau_{(s-q)}{}^{\alpha\beta}] = 0, \qquad \tau_{(s)}{}^{3\lambda} = \tau_{(s)}{}^{\lambda 3} - \zeta R b_\mu^\lambda \tau_{(s-q)}{}^{\mu 3}$$

$$- R^{-1} \partial W^{(s)} / \partial\zeta + \zeta b_\lambda^\lambda \partial W^{(s-q)} / \partial\zeta - \zeta^2 R K \partial W^{(s-2q)} / \partial\zeta = F_{33}^{33\,(0)} \tau_{(s+4p-4q)}^{33} + \dots$$
$$\dots + F_{33}^{33\,(k)} \tau_{(s+4p-4q-kq)}^{33} + \dots + F_{33}^{\lambda\mu\,(0)}\,[a_{\xi\mu} a_{\nu\lambda} \tau_{(s+2p-2q)}^{\xi\nu} -$$
$$- R\zeta\,(a_{\xi\mu} b_{\nu\lambda} + 2 a_{\nu\lambda} b_{\xi\mu})\, \tau_{(s+2p-3q)}^{\xi\nu}] + R^2 \zeta^2\,(b_{\xi\mu} b_{\nu\lambda} + a_{\nu\lambda} b_\xi^\alpha b_{\alpha\mu})\, \tau_{(s+2p-4q)}^{\xi\nu} -$$
$$- R^3 \zeta^3 b_\xi{}^\alpha b_{\alpha\mu} b_{\nu\lambda} \tau_{(s+2p-5q)}^{\xi\nu}] + \dots + F_{33}^{\lambda\mu\,(k)}\,[\cdots]_{(s-kq)} + \dots$$

$$^1/_2 R^{-1}\,[- \partial_\alpha W^{(s)} + \partial u_\alpha{}^{(s)} / \partial\zeta + R b_\alpha^\lambda u_\lambda{}^{(s-q)} - R\zeta b_\alpha^\lambda \partial u_\lambda{}^{(s-q)} / \partial\zeta] - {}^1/_2 \zeta b_\nu^\nu\,[\cdots]_{(s-q)} +$$
$$+ {}^1/_2 \zeta^2 R K\,[\cdots]_{(s-2q)} = F_{\alpha 3}^{\lambda 3\,(0)}\,[a_{\xi\lambda} \tau_{(s+2p-3q)}^{\xi 3} - 2\zeta R b_{\xi\lambda} \tau_{(s+2p-3q)}^{\xi 3} +$$
$$+ \zeta^2 R^2 b_\xi^\mu b_{\lambda\mu} \tau_{(s+2p-4q)}^{\xi 3}] + F_{\alpha 3}^{3\lambda\,(0)}\,[a_{\xi\lambda} \tau_{(s+2p-2q)}^{3\xi} - \zeta R b_{\xi\lambda} \tau_{(s+2p-3q)}^{3\xi}] + \dots$$
$$\dots + F_{\alpha 3}^{3\lambda\,(k)}\,[\cdots]_{(s-kq)} + \dots$$

$$^1/_2 R^{-1}\{\partial_\beta u_\alpha{}^{(s)} + \partial_\alpha u_\beta{}^{(s)} - 2R\Gamma_{\alpha\beta}^{\mu} u_\mu{}^{(s-p)} + 2R b_{\alpha\beta} W^{(s+q-2p)} -$$
$$- R\zeta\,[b_\beta^\lambda (\partial_\alpha u_\lambda{}^{(s-q)} - R\Gamma_{\alpha\lambda}^{\mu} u_\mu{}^{(s-p-q)} + R b_{\lambda\alpha} W^{(s-2p)}) + b_\alpha^\lambda (\partial_\beta u_\lambda{}^{(s-q)} -$$
$$- R\Gamma_{\beta\lambda}^{\mu} u_\mu{}^{(s-p-q)} + R b_{\lambda\beta} W^{(s-2p)})]\} - {}^1/_2 \zeta b_\lambda^\lambda \{\cdots\}_{(s-q)} + {}^1/_2 \zeta^2 R K\,\{\cdots\}_{(s-2q)} =$$
$$= F_{\alpha\beta}^{\lambda\mu\,(0)}\,[a_{\xi\mu} a_{\nu\lambda} \tau_{(s)}{}^{\xi\nu} - \zeta R\,(a_{\xi\mu} b_{\nu\lambda} + 2 a_{\nu\lambda} b_{\xi\mu})\, \tau_{(s-q)}{}^{\xi\nu} + \zeta^2 R^2\,(a_{\nu\lambda} b_\xi^\alpha b_{\alpha\mu} +$$
$$+ 2 b_{\xi\mu} b_{\nu\lambda})\, \tau_{(s-2q)}{}^{\xi\nu} - R^3 \zeta^3 b_\xi^\alpha b_{\alpha\mu} b_{\nu\lambda} \tau_{(s-3q)}{}^{\xi\nu}] + \dots + F_{\alpha\beta}^{\lambda\mu\,(k)}\,[\cdots]_{(s-kq)} + \dots$$
$$\dots + F_{\alpha\beta}^{33\,(0)} \tau_{(s+2p-2q)}^{33} + \dots + F_{\alpha\beta}^{33\,(k)} \tau_{(s+2p-2q-kq)}^{33} + \dots$$

The second consistent set of r is

$$\tau^{\alpha\beta} \to r = \varkappa + q, \qquad \tau^{\alpha 3} \to r = \varkappa + p, \qquad \tau^{33} \to r = \varkappa + 2p - q$$
$$u_\alpha \to r = \varkappa + q - p, \qquad W \to \varkappa < r < \varkappa + 2q - 2p \tag{4.8}$$

The leading system of equations of the corresponding sequence is (because of its cumbersomness, only the leading system is given)

$$\partial_\alpha \tau_{(0)}{}^{\alpha\beta} + \partial \tau_{(0)}{}^{3\beta} / \partial\zeta = 0, \quad \partial_\alpha \tau_{(0)}{}^{\alpha 3} + \partial \tau_{(0)}{}^{33} / \partial\zeta = 0, \quad c_{\lambda\beta} \tau_{(0)}{}^{\lambda\beta} = 0, \quad \tau_{(0)}{}^{3\lambda} = \tau_{(0)}{}^{\lambda 3} \tag{4.9}$$
$$\partial W^{(0)} / \partial\zeta = 0, \quad \partial u_\alpha{}^{(0)} / \partial\zeta = 0, \quad {}^1/_2 R^{-1}\,[\partial_\beta u_\alpha{}^{(0)} + \partial_\alpha u_\beta{}^{(0)}] = F_{\alpha\beta}^{\lambda\mu\,(0)} a_{\xi\mu} a_{\nu\lambda} \tau_{(0)}{}^{\xi\nu}$$

When $p = q$, the consistent set of values in (4.6) remains applicable. but the equations obtained in this case are different

$$\partial_\alpha \tau_{(s)}{}^{\alpha\beta} + \partial \tau_{(s)}{}^{3\beta} / \partial\zeta + R\,[\Gamma_{\mu\alpha}^{\alpha} \tau_{(s-p)}{}^{\mu\beta} + \Gamma_{\mu\alpha}^{\beta} \tau_{(s-p)}{}^{\mu\alpha}] - R b_\alpha^\beta \tau_{(s-p)}{}^{\alpha 3} = 0$$
$$\partial_\alpha \tau_{(s)}{}^{\alpha 3} + \partial \tau_{(s)}{}^{33} / \partial\zeta + R \Gamma_{\mu\alpha}^{\alpha} \tau_{(s-p)}{}^{\mu 3} + R b_{\alpha\beta} \tau_{(s-p)}{}^{\alpha\beta} = 0 \tag{4.10}$$
$$c_{\lambda\beta}\,[\tau_{(s)}{}^{\lambda\beta} - \zeta R b_\alpha^\lambda \tau_{(s-p)}{}^{\alpha\beta}] = 0, \qquad \tau_{(s)}{}^{3\lambda} = \tau_{(s)}{}^{\lambda 3} - \zeta R b_\mu^\lambda \tau_{(s-p)}{}^{\mu 3}$$

$$- R^{-1} \partial W^{(s)} / \partial\zeta + \zeta b_\lambda^\lambda \partial W^{(s-p)} / \partial\zeta - \zeta^2 R K \partial W^{(s-2p)} / \partial\zeta = F_{33}^{33\,(0)} \tau_{(s)}{}^{33} + \dots$$
$$\dots + F_{33}^{33\,(k)} \tau_{(s-kp)}^{33} + \dots + F_{33}^{\lambda\mu\,(0)}\,[a_{\xi\mu} a_{\nu\lambda} \tau_{(s)}{}^{\xi\nu} - \zeta R\,(a_{\xi\mu} b_{\nu\lambda} + 2 a_{\nu\lambda} b_{\xi\mu})\, \tau_{(s-p)}^{\xi\nu} +$$
$$+ R^2 \zeta^2\,(b_{\xi\mu} b_{\nu\lambda} + a_{\nu\lambda} b_\xi^\alpha b_{\alpha\mu})\, \tau_{(s-2p)}{}^{\xi\nu} - R^3 \zeta^3 b_\xi^\alpha b_{\alpha\mu} b_{\nu\lambda} \tau_{(s-3p)}{}^{\xi\nu}] + \dots$$
$$\dots + F_{33}^{\lambda\mu\,(k)}\,[\cdots]_{(s-kp)} + \dots$$

$$\tfrac{1}{2}R^{-1}[-\partial_\alpha W^{(s)} + \partial u_\alpha{}^{(s)}/\partial\zeta + Rb^\lambda_\alpha u_\lambda{}^{(s-p)} - R\zeta b^\lambda_\alpha \partial u_\lambda{}^{(s-p)}/\partial\zeta] - \tag{4.10 cont.}$$
$$-\tfrac{1}{2}\zeta b^\nu_\nu[\cdots]_{(s-p)} + \tfrac{1}{2}\zeta^2 RK[\cdots]_{(s-2p)} = F_{\alpha 3}^{\lambda 3\,(0)}[a_{\xi\lambda}\tau_{(s)}{}^{\xi 3} - 2\zeta R b_{\xi\lambda}\tau_{(s-p)}{}^{\xi 3} +$$
$$+\zeta^2R^2 b^\mu_\xi b_{\lambda\mu}\tau_{(s-2p)}{}^{\xi 3}] + F_{\alpha 3}^{3\lambda\,(0)}[a_{\xi\lambda}\tau_{(s)}{}^{3\xi} - \zeta R b_{\xi\lambda}\tau_{(s-p)}{}^{3\xi}] + \dots$$
$$+\dots F_{\alpha 3}^{\lambda 3\,(k)}[\cdots]_{(s-kp)} + \dots$$

$$\tfrac{1}{2}R^{-1}\{\partial_\beta u_\alpha{}^{(s)} + \partial_\alpha u_\beta{}^{(s)} - 2R\Gamma^\mu_{\alpha\beta}u_\mu{}^{(s-p)} + 2Rb_{\alpha\beta}W^{(s-p)} - R\zeta[b^\lambda_\beta(\partial_\lambda u_\lambda{}^{(s-p)} -$$
$$-R\Gamma^\mu_{\alpha\lambda}u_\mu{}^{(s-2p)} + Rb_{\lambda\alpha}W^{(s-2p)}) + b^\lambda_\alpha(\partial_\beta u_\lambda{}^{(s-p)} - R\Gamma^\mu_{\beta\lambda}u_\mu{}^{(s-2p)} + Rb_{\lambda\beta}W^{(s-2p)})]\} -$$
$$-\tfrac{1}{2}\zeta b^\lambda_\lambda\{\cdots\}_{(s-p)} + \tfrac{1}{2}\zeta^2RK\{\cdots\}_{(s-2p)} = F_{\alpha\beta}^{\lambda\mu\,(0)}[a_{\xi\mu}a_{\nu\lambda}\tau^{\xi\nu}_{(s)} -$$
$$-\zeta R(a_{\xi\mu}b_{\nu\lambda} + 2a_{\nu\lambda}b_{\xi\mu})\tau_{(s-p)}{}^{\xi\nu} + \zeta^2R^2(a_{\nu\lambda}b^\alpha_\xi b_{\alpha\mu} + 2b_{\xi\mu}b_{\nu\lambda})\tau_{(s-2p)}{}^{\xi\nu} -$$
$$-R^3\zeta^3 b^\alpha_\xi b_{\alpha\mu}b_{\nu\lambda}\tau_{(s-3p)}{}^{\xi\nu}] + \dots + F_{\alpha\beta}^{\lambda\mu\,(k)}[\cdots]_{(s-kp)} + \dots + F_{\alpha\beta}^{33\,(0)}\tau_{(s)}{}^{33} + \dots$$
$$\dots + F_{\alpha\beta}^{33\,(k)}\tau_{(s-kp)}{}^{33}$$

It is easily shown that the state of stress defined by the first approximation in Equations (4.7), (4.9) and (4.10) is equivalent to that obtained from classical theory for large values of the exponent of variation.

5. Auxiliary iterative process. Consider a state of stress having different variations in the ϑ^1 and ϑ^2 directions. For definiteness, suppose that the greater variation takes place in the ϑ^1 direction. Assume that $K_{(1)} = h^{*-1} = \eta$, while $K_{(2)} = \eta^0$.

We will show that the state of stress in this case is essentially different from the case considered above. Let

$$\vartheta^1 = Rh^*\xi^1, \qquad \vartheta^2 = R\xi^2, \qquad \vartheta^3 = h\zeta \tag{5.1}$$

We now seek a solution to (4.3) in the form given in (4.5). Then there are two consistent sets of r. The first one is

$$(\tau^{11}, \tau^{22}, \tau^{33}, \tau^{31}, \tau^{13}) \to r = \varkappa - 1, \quad (\tau^{12}, \tau^{21}, \tau^{23}, \tau^{32}) \to r = \varkappa$$
$$(u_1, W) \to r = \varkappa - 2, \quad u_2 \to r = \varkappa - 1 \tag{5.2}$$

This set corresponds to the following system of equations:

$$\partial_1\tau^{11}_{(s)} + \partial_2\tau^{21}_{(s)} + R[(2\Gamma^1_{11} + \Gamma^2_{12})\tau_{(s-1)}{}^{11} + \Gamma^1_{22}\tau_{(s-1)}{}^{22}] + R[(2\Gamma^1_{12} + \Gamma^2_{22})\tau^{21}_{(s)} + \tag{5.3}$$
$$+\Gamma^1_{21}\tau^{12}_{(s)}] - Rb^1_1\tau^{13}_{(s-1)} - Rb^1_2\tau^{23}_{(s)} + \partial\tau_{(s)}{}^{31}/\partial\zeta = 0$$

$$\partial_1\tau_{(s)}{}^{12} + \partial_2\tau_{(s-2)}{}^{22} + R[(\Gamma^1_{11} + 2\Gamma^2_{21})\tau_{(s-1)}{}^{12} + \Gamma^2_{12}\tau_{(s-1)}{}^{21} +$$
$$+\tau_{(s-2)}{}^{22}(\Gamma^2_{21} + 2\Gamma^2_{22}) + \Gamma^2_{11}\tau^{11}_{(s-2)}] - Rb^2_1\tau^{13}_{(s-2)} - Rb^2_2\tau^{23}_{(s-1)} + \partial\tau_{(s)}{}^{32}/\partial\zeta = 0$$

$$\partial_1\tau_{(s)}{}^{13} + \partial_2\tau_{(s)}{}^{23} + R(\Gamma^1_{11} + \Gamma^2_{12})\tau_{(s-1)}{}^{13} + R\tau_{(s)}{}^{23}(\Gamma^1_{21} + \Gamma^2_{22}) +$$
$$+R(b_{11}\tau_{(s-1)}{}^{11} + b_{22}\tau_{(s-1)}{}^{22}) + R(b_{12}\tau_{(s)}{}^{12} + b_{21}\tau_{(s)}{}^{21}) + \partial\tau_{(s)}{}^{33}/\partial\zeta = 0$$

$$c_{12}\tau_{(s)}{}^{12} + c_{21}\tau_{(s)}{}^{21} - \zeta R[c_{12}(b^1_1\tau^{12}_{(s-1)} + b^1_2\tau^{22}_{(s-2)}) + c_{21}(b^2_1\tau^{11}_{(s-1)} + b^2_2\tau^{21}_{(s-1)})] = 0$$

$$\tau^{31}_{(s)} = \tau^{13}_{(s)} - \zeta R[b^1_1\tau^{13}_{(s-1)} + b^1_2\tau^{23}_{(s)}], \qquad \tau^{32}_{(s)} = \tau^{23}_{(s)} - \zeta R[b^2_1\tau^{13}_{(s-2)} + b^2_2\tau^{23}_{(s-1)}]$$

$$-R^{-1}\partial W^{(s)}/\partial\zeta + \zeta b^\lambda_\lambda\partial W^{(s-1)}/\partial\zeta - \zeta^2RK\partial W^{(s-2)}/\partial\zeta = F_{33}^{33\,(0)}\tau^{33}_{(s)} + \dots$$
$$\dots + F_{33}^{33\,(k)}\tau^{33}_{(s-k)} + \dots + F_{33}^{\lambda\mu\,(0)}[a_{\nu\mu}a_{\nu\lambda}\tau_{(s)}{}^{\nu\nu} - \zeta R(a_{\nu\mu}b_{\nu\lambda} + 2a_{\nu\lambda}b_{\nu\mu})\tau_{(s-1)}{}^{\nu\nu} +$$
$$+\zeta^2R^2(a_{\nu\lambda}b^\alpha_\nu b_{\alpha\mu} + 2b_{\nu\mu}b_{\nu\lambda})\tau_{(s-2)}{}^{\nu\nu} - R^3\zeta^3 b^\alpha_\nu b_{\alpha\mu}b_{\nu\lambda}\tau_{(s-3)}{}^{\nu\nu} -$$

$$- \zeta R (a_{\nu\mu} b_{\eta\lambda} + 2a_{\eta\lambda} b_{\nu\mu}) \tau_{(s)}{}^{\nu\eta} + R^2\zeta^2 (a_{\eta\lambda} b_\nu^\alpha b_{\alpha\mu} + 2b_{\nu\mu} b_{\eta\lambda}) \tau_{(s-1)}{}^{\nu\eta} - \quad (5.3)\ \text{cont.}$$

$$- R^3\zeta^3 b_\nu^\alpha b_{\alpha\mu} b_{\eta\lambda} \tau_{(s-2)}^{\nu\eta}] + \ldots + F_{33}^{\lambda\mu\,(k)} [\cdots]_{(s-k)} + \ldots \qquad (\eta \neq \nu)$$

$$\tfrac{1}{2}R^{-1} [- \partial_1 W^{(s)} + R b_1^2 u_2{}^{(s)} + \partial u_1{}^{(s)} / \partial\zeta + R b_1^1 u_1{}^{(s-1)} - \zeta R (b_1^1 \partial u_1{}^{(s-1)} / \partial\zeta +$$

$$+ b_1^2 \partial u_2{}^{(s)} / \partial\zeta)] - \tfrac{1}{2}\zeta b_\lambda^\lambda [\cdots]_{(s-1)} + \tfrac{1}{2}\zeta^2 R K [\cdots]_{(s-2)} = F_{13}^{\lambda 3\,(0)} [a_{1\lambda} \tau_{(s)}{}^{13} -$$

$$- 2\zeta R b_{1\lambda} (\tau_{(s)}^{23} + \tau_{(s-1)}^{13}) + \zeta^2 R^2 b_1^\alpha b_{\alpha\lambda} (\tau_{(s-2)}^{13} + \tau_{(s-1)}^{23}) - \zeta R b_{2\lambda} \tau_{(s)}{}^{32}] +$$

$$+ F_{13}^{3\lambda\,(0)} [a_{1\lambda} \tau_{(s)}^{31} - \zeta R b_{1\lambda} \tau_{(s-1)}^{31}] + \ldots + F_{13}^{\lambda 3\,(k)} [\cdots]_{(s-k)} + \ldots$$

$$\tfrac{1}{2}R^{-1} [\partial u_2^{(s)} / \partial\zeta - \partial_2 W^{(s-2)} + R b_2^1 u_1{}^{(s-2)} + R b_2^2 u_2{}^{(s-1)} -$$

$$- R\zeta (b_2^1 \partial u_1^{(s-2)} / \partial\zeta + b_2^2 \partial u_2^{(s-1)} / \partial\zeta)] - \tfrac{1}{2}\zeta b_\lambda^\lambda [\cdots]_{(s-1)} + \tfrac{1}{2}\zeta^2 R K [\cdots]_{(s-2)} =$$

$$= F_{23}^{\lambda 3\,(0)} [a_{1\lambda} \tau_{(s-1)}^{13} - 2\zeta R (b_{1\lambda} \tau_{(s-2)}^{13} + b_{2\lambda} \tau_{(s-1)}^{23}) + a_{2\lambda} \tau_{(s)}^{23} +$$

$$+ \zeta^2 R^2 (b_1^\alpha b_{\alpha\lambda} \tau_{(s-3)}^{13} + b_2^\alpha b_{\alpha\lambda} \tau_{(s-2)}^{23})] + F_{23}^{3\lambda\,(0)} [a_{1\lambda} \tau_{(s-1)}^{31} -$$

$$- \zeta R (b_{1\lambda} \tau_{(s-2)}^{31} + b_{2\lambda} \tau_{(s-2)}^{32}) + a_{2\lambda} \tau_{(s-1)}^{32}] + \ldots + F_{23}^{\lambda 3\,(k)} [\cdots]_{(s-k)} + \ldots$$

$$R^{-1} \{\partial_1 u_1{}^{(s)} - R (\Gamma_{11}^1 u_1{}^{(s-1)} + \Gamma_{11}^2 u_2{}^{(s)}) + b_{11} R W^{(s-1)} - \zeta R [b_1^1 (\partial_1 u_1{}^{(s-1)} - R (\Gamma_{11}^1 u_1{}^{(s-2)} +$$

$$+ \Gamma_{11}^2 u_2{}^{(s-1)}) + R b_{11} W^{(s-2)}) + b_1^2 (\partial_1 u_2{}^{(s)} - R (\Gamma_{12}^1 u_1{}^{(s-2)} + \Gamma_{12}^2 u_2{}^{(s-1)}) + R b_{21} W^{(s-2)})]\} -$$

$$- \zeta b_\lambda^\lambda \{\cdots\}_{(s-1)} + \zeta^2 R K \{\cdots\}_{(s-2)} = F_{11}^{33\,(0)} \tau_{(s)}^{33} + \ldots + F_{11}^{33\,(k)} \tau_{(s-k)}^{33} + \ldots$$

$$\ldots + F_{11}^{\lambda\mu\,(0)} [a_{\nu\lambda} a_{\nu\mu} \tau_{(s)}^{\nu\nu} - \zeta R (a_{\nu\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\nu\mu}) \tau_{(s-1)}^{\nu\nu} +$$

$$+ \zeta^2 R^2 (2b_{\nu\lambda} b_{\nu\mu} + a_{\nu\lambda} b_\nu^\xi b_{\mu\xi}) \tau_{(s-2)}^{\nu\nu} - \zeta^3 R^3 b_{\nu\lambda} b_\nu^\xi b_{\mu\xi} \tau_{(s-3)}^{\nu\nu} - \zeta R (a_{\eta\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\eta\mu}) \tau_{(s)}^{\eta\nu} +$$

$$+ \zeta^2 R^2 (2b_{\nu\lambda} b_{\eta\mu} + a_{\nu\lambda} b_\eta^\xi b_{\mu\xi}) \tau_{(s-1)}^{\eta\nu} - \zeta^3 R^3 b_{\nu\lambda} b_\eta^\xi b_{\mu\xi} \tau_{(s-2)}^{\eta\nu}] + \ldots + F_{11}^{\lambda\mu\,(k)} [\cdots]_{(s-k)} + \ldots$$

$$\tfrac{1}{2}R^{-1} \{\partial_2 u_1{}^{(s-2)} - 2R (\Gamma_{12}^1 u_1{}^{(s-2)} + \Gamma_{12}^2 u_2{}^{(s-1)}) + \partial_1 u_2{}^{(s)} + 2b_{12} R W^{(s-2)} -$$

$$- \zeta R [b_2^1 (\partial_1 u_1{}^{(s-2)} - R (\Gamma_{11}^1 u_1{}^{(s-3)} + \Gamma_{11}^2 u_2{}^{(s-2)}) + R b_{11} W^{(s-3)}) +$$

$$+ b_2^2 (\partial_1 u_2{}^{(s-1)} - R (\Gamma_{12}^1 u_1{}^{(s-3)} + \Gamma_{12}^2 u_2{}^{(s-2)}) + R b_{21} W^{(s-3)}) +$$

$$+ b_1^1 (\partial_2 u_1{}^{(s-3)} - R (\Gamma_{12}^1 u_1{}^{(s-3)} + \Gamma_{12}^2 u_2{}^{(s-2)}) + R b_{12} W^{(s-3)}) +$$

$$+ b_1^2 (\partial_2 u_2{}^{(s-1)} - R (\Gamma_{22}^1 u_1{}^{(s-3)} + \Gamma_{22}^2 u_2{}^{(s-3)}) + R b_{22} W^{(s-3)})]\} - \tfrac{1}{2}\zeta b_\lambda^\lambda \{\cdots\}_{(s-1)} +$$

$$+ \tfrac{1}{2}\zeta^2 R K \{\cdots\}_{(s-2)} = F_{12}^{33\,(0)} \tau_{(s-1)}^{33} + \ldots + F_{12}^{33\,(k)} \tau_{(s-k-1)}^{33} + \ldots$$

$$\ldots + F_{12}^{\lambda\mu\,(0)} [a_{\nu\lambda} a_{\nu\mu} \tau_{(s-1)}^{\nu\nu} - \zeta R (a_{\nu\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\nu\mu}) \tau_{(s-2)}^{\nu\nu} + \zeta^2 R^2 (2b_{\nu\lambda} b_{\nu\mu} + a_{\nu\lambda} b_\nu^\xi b_{\mu\xi}) \tau_{(s-3)}^{\nu\nu} -$$

$$- \zeta^3 R^3 b_{\nu\lambda} b_\nu^\xi b_{\mu\xi} \tau_{(s-4)}^{\nu\nu} + a_{\nu\lambda} a_{\eta\mu} \tau_{(s)}^{\eta\nu} - \zeta R (a_{\eta\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\eta\mu}) \tau_{(s-1)}^{\eta\nu} +$$

$$+ \zeta^2 R^2 (2b_{\nu\lambda} b_{\eta\mu} + a_{\nu\lambda} b_\eta^\xi b_{\mu\xi}) \tau_{(s-2)}^{\eta\nu} - \zeta^3 R^3 b_{\nu\lambda} b_\eta^\xi b_{\mu\xi} \tau_{(s-3)}^{\eta\nu}] + \ldots + F_{12}^{\lambda\mu\,(k)} [\cdots]_{(s-k)} + \ldots$$

$$R^{-1} \{\partial_2 u_2{}^{(s)} - R (\Gamma_{22}^1 u_1{}^{(s-1)} + \Gamma_{22}^2 u_2{}^{(s)}) + b_{22} R W^{(s-1)} - R\zeta [b_2^1 (\partial_2 u_1{}^{(s-2)} +$$

$$+ R (\Gamma_{12}^1 u_1{}^{(s-2)} + \Gamma_{12}^2 u_2{}^{(s-1)}) + R b_{12} W^{(s-2)}) + b_2^2 (\partial_2 u_2{}^{(s-1)} + R (\Gamma_{22}^1 u_1{}^{(s-2)} +$$

$$+ \Gamma_{22}^2 u_2{}^{(s-1)}) + R b_{22} W^{(s-2)})] - \zeta b_\lambda^\lambda \{\cdots\}_{(s-1)} + \zeta^2 R K \{\cdots\}_{(s-2)} = F_{22}^{33\,(0)} \tau_{(s)}{}^{33} + \ldots$$

$$\ldots + F_{22}^{33\,(k)} \tau_{(s-k)}^{33} + \ldots + F_{22}^{\lambda\mu\,(0)} [a_{\nu\lambda} a_{\nu\mu} \tau_{(s)}^{\nu\nu} - \zeta R (a_{\nu\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\nu\mu}) \tau_{(s-1)}^{\nu\nu} +$$

$$+ \zeta^2 R^2 (2b_{\nu\lambda} b_{\nu\mu} + a_{\nu\lambda} b_\nu^\xi b_{\mu\xi}) \tau_{(s-2)}^{\nu\nu} - \zeta^3 R^3 b_{\nu\lambda} b_\nu^\xi b_{\mu\xi} \tau_{(s-3)}^{\nu\nu} - \zeta R (a_{\eta\mu} b_{\nu\lambda} + 2a_{\nu\lambda} b_{\eta\mu}) \tau_{(s)}{}^{\eta\nu} +$$

$$+ \zeta^2 R^2 (2b_{\nu\lambda} b_{\eta\mu} + a_{\nu\lambda} b_\eta^\xi b_{\mu\xi}) \tau_{(s-1)}^{\eta\nu} - \zeta^3 R^3 b_{\nu\lambda} b_\eta^\xi b_{\mu\xi} \tau_{(s-2)}^{\eta\nu}] + \ldots + F_{22}^{\lambda\mu\,(k)} [\cdots]_{(s-k)} + \ldots$$

$$(\eta \neq \nu)$$

When an orthogonal coordinate system is used for the middle surface of the shell, then

$$a_{ij}=0, \qquad \Gamma_{ij}^{r}=0 \qquad (i\neq j\neq r\neq i)$$

and the system of Equations (5.3) becomes considerably simplified.

The second consistent set of r is given by

$$(\tau^{11},\ \tau^{22},\ \tau^{33},\ \tau^{13},\ \tau^{31})\rightarrow r=\varkappa, \quad (\tau^{12},\ \tau^{21},\ \tau^{23},\ \tau^{32})\rightarrow r=\quad -1$$

$$(u_1,\ W)\rightarrow r=\varkappa-1, \quad u_2\rightarrow r=\varkappa-2$$

Corresponding to this set there is also a sequence of systems of equations, obtained in the manner described above. In view of their cumbersomeness, only the leading system of equations is given (5.4)

$$\partial_1\tau_{(0)}{}^{11}+\partial\tau_{(0)}{}^{31}/\partial\zeta=0, \quad \partial_1\tau_{(0)}{}^{12}+\partial_2\tau_{(0)}{}^{22}+R\,(\Gamma_{21}^{1}+2\Gamma_{22}^{2})\,\tau_{(0)}{}^{22}+R\Gamma_{11}^{2}\tau_{(0)}{}^{11}-$$

$$-Rb_1^2\tau_{(0)}{}^{13}+\partial\tau_{(0)}{}^{32}/\partial\zeta=0, \quad \tau_{(0)}{}^{31}=\tau_{(0)}{}^{13}, \quad \partial_1\tau_{(0)}{}^{13}+\partial\tau_{(0)}{}^{33}/\partial\zeta=0$$

$$\tau_{(0)}{}^{32}=\tau_{(0)}{}^{23}-\zeta Rb_1^2\tau_{(0)}{}^{13}, \quad c_{12}\tau_{(0)}{}^{12}+c_{21}\tau_{(0)}{}^{21}-\zeta R\,[c_{21}b_1^2\tau_{(0)}{}^{11}+c_{12}b_2^1\tau_{(0)}{}^{22}]=0$$

$$-R^{-1}\partial W_{(0)}/\partial\zeta=F_{33}^{33}\tau_{(0)}{}^{33}+F_{33}^{\lambda\mu\,(0)}a_{1\mu}a_{1\lambda}\tau_{(0)}{}^{11}+F_{33}^{\lambda\mu\,(0)}a_{2\mu}a_{2\lambda}\tau_{(0)}{}^{22}$$

$$1/_2R^{-1}\,[-\partial_1W^{(0)}+\partial u_1^{(0)}/\partial\zeta]=F_{13}^{3\lambda\,(0)}a_{1\lambda}\tau_{(0)}{}^{13}+F_{31}^{3\lambda\,(0)}a_{1\lambda}\tau_{(0)}{}^{31}$$

$$1/_2R^{-1}\,[-\partial_2W^{(0)}+\partial u_2{}^{(0)}/\partial\zeta+Rb_2^1u_1{}^{(0)}-R\zeta b_2^1\partial u_1{}^{(0)}/\partial\zeta]=$$

$$=F_{32}^{3\lambda\,(0)}\,(a_{2\lambda}\tau_{(0)}{}^{23}-2\zeta Rb_{1\lambda}\tau_{(0)}{}^{13})+F_{23}^{3\lambda\,(0)}\,(a_{2\lambda}\tau_{(0)}{}^{32}-\zeta Rb_{1\lambda}\tau_{(0)}{}^{31})$$

$$R^{-1}\partial_1u_1^{(0)}=F_{11}^{33(0)}\tau_{(0)}{}^{33}+F_{11}^{\lambda\mu(0)}a_{\nu\lambda}a_{\nu\mu}\tau_{(0)}{}^{\nu\nu}.$$

$$1/_2R^{-1}\,[\partial_2u_1{}^{(0)}-2R\Gamma_{12}^{1}u_1{}^{(0)}+\partial_1u_2{}^{(0)}+2b_{12}RW^{(0)}-\zeta Rb_2^1\partial_1u_1{}^{(0)}]=F_{12}^{\lambda\mu\,(1)}a_{\nu\lambda}a_{\nu\mu}\tau_{(0)}{}^{\nu\nu}-$$

$$-\zeta RF_{12}^{\lambda\mu\,(0)}\,(a_{\nu\mu}b_{\nu\lambda}+2a_{\nu\lambda}b_{\nu\mu})\,\tau_{(0)}{}^{\nu\nu}+F_{12}^{\lambda\mu}a_{\nu\lambda}a_{\eta\mu}\tau_{(0)}{}^{\eta\nu}+F_{12}^{33\,(1)}\tau_{(0)}{}^{33} \qquad (\eta\neq\nu)$$

$$F_{22}^{33\,(0)}\tau_{(0)}{}^{33}+F_{22}^{\lambda\mu\,(0)}a_{\nu\lambda}a_{\nu\mu}\tau_{(0)}{}^{\nu\nu}=0$$

If, in the preceding Equations (5.4), we let

$$F_{33}^{33\,(0)}=1/E, \qquad F_{33}^{\lambda\mu\,(0)}=-\sigma/Ea^{\lambda\mu}, \qquad F_{31}^{3\lambda\,(0)}=[(1+\sigma)/E]\,\delta_1^{\lambda}$$

$$F_{32}^{3\lambda\,(0)}=[(1+\sigma)/E]\,\delta_2^{\lambda}$$

$$F_{11}^{33\,(0)}=-\sigma/Ea_{11}, \quad E_{12}^{\lambda\mu\,(0)}=[(1+\sigma)/E]\ \delta_1^{\mu}\delta_2^{\mu}$$

$$F_{11}^{\lambda\mu\,(0)}=[(1+\sigma)/E]\ \delta^{\lambda}\delta_1^{\mu}-\sigma/Ea^{\lambda\mu}a_{11}$$

$$F_{12}^{\lambda\mu\,(1)}=2\sigma/E\zeta Rb_{12}a^{\lambda\mu}, \qquad F_{12}^{33\,(1)}=2\sigma/E\zeta Rb_{12}, \qquad F_{22}^{33\,(0)}=-\sigma/Ea_{22}$$

$$F_{22}^{\lambda\mu\,(0)}=[(1+\sigma)/E]\,\delta_2^{\lambda}\delta_2^{\mu}-\sigma/Ea^{\lambda\mu}a_{22}, \qquad a_{ij}=0 \quad (\text{for}\quad i\neq j)$$

then these equations reduce to the isotropic shell equations [1].

For the zeroth approximation, the second, fourth, sixth, ninth and eleventh equations in (5.3) comprise a self-contained subsystem in terms of $\tau_{(0)}^{12}$, $\tau_{(0)}^{21}$, $\tau_{(0)}^{23}$, $\tau_{(0)}^{32}$, and $u_2^{(0)}$ corresponding essentially to the classical problem concerning torsion of an anisotropic bar about the ξ^2-axis. The first, third, fourth, seventh, eighth, tenth and twelfth equations in (5.4) comprise a self-contained subsystem in terms of $\tau_{(0)}^{11}$, $\tau_{(0)}^{22}$, $\tau_{(0)}^{33}$ $\tau_{(0)}^{13}$, $\tau_{(0)}^{31}$, $u_1{}^{(0)}$ and $W^{(0)}$. These equations correspond essentially to the plane strain problem in the $\xi^1\zeta$ plane.

The stress and strain states for an anisotropic shell may now be written as the sum of two stress and strain states, obtained through the fundamental and auxiliary iterative processes, and we require that the stresses thus obtained satisfy the boundary conditions (2.6). In addition to these boundary conditions, the stresses and displacements thus obtained must also satisfy the boundary constraints. Evidently, we can combine the fundamental and auxiliary processes so as to satisfy the shell boundary conditions to any desired accuracy. This problem has been studied in detail in connection with plates [2], but requires separate examination for shells.

The author is grateful to S.A. Ambartsumian for his valuable suggestions, and thanks A.L. Gol'denvenveizer for reviewing this work.

BIBLIOGRAPHY

1. Gol'denveizer, A.L., Postroenie pribizhennoi teorii obolochek pri pomoshchi asimptoticheskogo integrirovaniia uravnenii teorii uprugosti (Derivation of an approximate theory of shells by means of asymptotic integration of the equations of the theory of elasticity). *PMM* Vol. 27, № 4, 1963.
2. Gol'denveizer, A.L., Postroenie priblizhennoi teorii izgiba plastinki metodom asimptoticheskogo integrirovaniia uravnenii teorii uprugosti (Derivation of an approximate theory of a plate bending by the method of asymptotic integration of the equations of the theory of elasticity). *PMM* Vol.26, № 4, 1962.
3. Green, A.E. and Zerna, W., Theoretical Elasticity, Oxford Clarendon Press, 1954.
4. Kochin, N.E., Vektornoe ischislenie i nachala tenzornogo ischisleniia (Vector Calculus and Introductory Tensor Calculus). M.-L., Gostekhizdat, 1934.
5. Ambartsumian, S.A., Teoriia anizotropnykh obolochek (Theory of Anisotropic Shells). M., Fizmatgiz, 1961.
6. Gol'denveizer, A.L., Teoriia uprugikh tonkikh obolochek (Theory of Thin Elastic Shells). M. Gostekhizdat, 1953.

Translated by H.H.